Polymer in a Random Flow with Mean Shear

We discuss here the statistics of polymers placed in a chaotic flow with a relatively large mean shear, that is the flow of the type correspondent to the elastic turbulence experiments by Groisman & Steinberg (2000,2001,2004). We assume that the effect of velocity fluctuations is stronger than that related to thermal noise, and that polymers are essentially elongated so that the polymer orientation is well defined. The main body of the orientational fluctuations occur in a neighborhood of a special direction preferred by the shear. Sometimes the typical fluctuations are interrupted by flips, in which the polymer orientation is reversed. The task of this study is to describe statistics of the angular orientation and tumbling time as well as statistics of the polymer extension. We establish the main features of the PDFs for the objects.

Model. We consider a single polymer molecule advected by a chaotic/turbulent flow (i.e. the polymer moves along a Lagrangian trajectory of the flow) and is stretched by velocity inhomogeneity. The polymer stretching is characterized by the molecule's end-to-end separation vector, $\boldsymbol{R}$, satisfying the following dumb-bell-like equation:

$$\partial_t R_i = R_j \nabla_j v_i - \gamma(R) R_i + \zeta_i \,. \qquad (1)$$

Here γ is the polymer relaxation rate and ζ_i is the Langevin force. The velocity gradient $\nabla_j v_i$ is taken at the molecule position. The velocity difference between the polymer end points is approximated in Eq. (1) by the first term of its Taylor expansion in the end-to-end vector. It is justified if the polymer size is less than the velocity correlation length. The relaxation rate γ in Eq. (1) is a function of the extension R which varies from zero upto a maximum value $R_{\max}$ corresponding to a fully stretched polymer. We assume that the relaxation is Hookean for $R \ll R_{\max}$, i.e. $\gamma(R)$ is well approximated by a constant $\gamma(0)$ there, while it diverges (the polymer becomes stiff) for $R \to R_{\max}$.

We focus on the situation in which the effect of velocity fluctuations is stronger than that of thermal fluctuations, so that the Langevin force $\boldsymbol{\zeta}$ in Eq. (1) can be neglected. We consider the case in which the steady shear flow is accompanied by weaker random velocity fluctuations. This is also the setting realized in the elastic turbulence experiments by Groisman & Steinberg (2000,2001,2004). We choose the coordinate frame associated with the shear flow, as shown in Fig. 1, where the mean flow is characterized by the shear velocity $(sy, 0, 0)$ and s is positive. Then the polymer end-to-end vector $\boldsymbol{R}$ is conveniently parameterized by the spherical angles ϕ and θ: $R_x = R\cos\theta\cos\phi$, $R_y = R\cos\theta\sin\phi$, $R_z = R\sin\theta$.

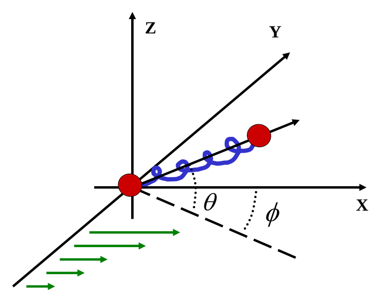

Figure 1: Scheme explaining polymer orientation geometry.

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$$\partial_t \phi = -s\sin^2\phi + \xi_\phi \,, \qquad (2)$$

$$\partial_t \theta = -s\sin\phi\cos\phi\sin\theta\cos\theta + \xi_\theta \,, \qquad (3)$$

$$\partial_t \ln R = -\gamma(R) + s\cos^2\theta\cos\phi\sin\phi + \xi_\parallel \,, \qquad (4)$$

where ξ_ϕ, ξ_θ and $\xi_\parallel$ are random variables related to the fluctuating component of the velocity gradient.

Angular statistics. The statistics of the velocity fluctuations is assumed to be homogeneous in time. In a statistically stationary velocity field, the angular statistics is stationary as well, being characterized by the joint PDF, $\mathcal{P}(\phi,\theta)$, which is a periodic function of the angles with the period π for both ϕ and θ. Thus, it is sufficient to consider $\mathcal{P}(\phi,\theta)$ within the following bounded domain (torus), $-\pi/2 < \phi,\theta < \pi/2$. According to Eqs. (2,3), $\mathcal{P}(\phi,\theta)$ is symmetric with respect to θ but it is not symmetric with respect to ϕ. Therefore, the average value of ϕ, $\phi_t = \langle\phi\rangle$, is non-zero. In our setting, ϕ_t is positive. The value of ϕ_t can be estimated by balancing the deterministic and stochastic terms on the right hand side of Eq. (2). The weakness of the random term in comparison with s implies $\phi_t \ll 1$. The same quantity ϕ_t estimates typical fluctuations of ϕ about its mean value. It immediately follows that the typical value of θ fluctuations is estimated by ϕ_t as well.

It is natural to expect that the Lagrangian velocity correlation time is $\bar{\lambda}^{-1} = [s\phi_t]^{-1}$, that is also a characteristic time of the ξ_ϕ and ξ_θ variations. Then, comparing the left hand sides of Eqs. (2,3) with the first terms on their right hand sides (for $\phi,\theta \ll 1$), one concludes that the angular correlation time can be estimated by the same quantity $\bar{\lambda}^{-1} = (s\phi_t)^{-1}$. Next, equating the terms on the right hand sides of Eqs. (2,3), one derives $\xi_\phi \sim \xi_\theta \sim s\phi_t^2 \ll s$. The last inequality reflects the assumed weakness of the velocity gradient fluctuations compared to the shear rate, s.

Tails of the angular PDFs. One finds two different contributions to the PDF tail: one is related to the deterministic motion while the other is associated with the stochastic evolution in the domain, $|\phi| < \phi_t, |\theta| \gg \phi_t$. For $1 \gg |\theta| \gg \phi_t$, both contributions are algebraic, $\propto |\theta|^{-2}$ and $\propto |\theta|^{-a}$, respectively. The deterministic contribution, $\propto |\theta|^{-2}$, dominates if $a > 2$, while the stochastic contribution, $\propto |\theta|^{-a}$, dominates otherwise.

Tumbling time statistics. The deterministic process, which defines the polymer turn (because ϕ changes essentially only during the deterministic part of the dynamics), is faster than the stochastic wandering taking place at small angles, $|\phi|,|\theta| \sim \phi_t$. Therefore it is convenient to define the tumbling time, τ, as the time separating two subsequent crossings in ϕ of the special angle $\pm\pi/2$, in the middle of the deterministic domain. Since the major contribution to τ originates from the stochastic wandering in the ϕ_t-narrow vicinity of $\phi = 0$, the position of the τ-PDF maximum and its width are both estimated by the correlation time $(s\phi_t)^{-1}$, because this is the only relevant characteristic time of the stochastic evolution.

Considering the PDF tail for $\tau \gg \bar{\lambda}^{-1}$, one observes that if a flip does not occur for a long time, then this delay can be interpreted in terms of the large number, $\bar{\lambda}\tau$, of independent unsuccessful attempts to pass (clock-wise in ϕ) the stochastic domain $|\phi| < \phi_t$. The probability of the delayed flip is given by the product of the probabilities of these $\bar{\lambda}\tau$ events, resulting in $\ln P_\tau \sim -\bar{\lambda}\tau$, for $\tau \gg \bar{\lambda}^{-1}$.

Polymer extension statistics. To illustrate our generic analytical results, that are explained in details in [2], we plot in Fig. 2 four graphs of the extension PDF obtained by numerical simulations in a simple model flow done with $\gamma(R) = \gamma(0)/(1-R^2/R_{\max}^2)$, corresponding to the so-called FENE-P model of the polymer elasticity.

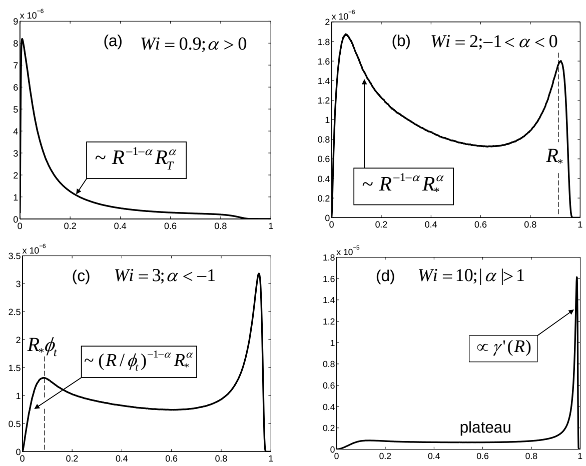

Figure 2: PDF of the polymer extension, R, measured in the units of maximal extension, for different values of the Weissenberg number, Wi, obtained from numerical simulations explained in [2].

References

[1] M. Chertkov, Polymer Stretching by Turbulence, *Phys. Rev. Lett.* **84** 4761 (2000).

[2] M. Chertkov, I. Kolokolov, V. Lebedev and K. Turitsyn, Polymer Statistics in a Random Flow with Mean Shear, to appear in JFM 2005.

Contact Information: Michael Chertkov – Complex systems Group and Center for Nonlinear Studies, Theoretical Division, Los Alamos National Laboratory, MS-B258, Los Alamos, NM, 87545, Phone: (505) 665-8119, email: chertkov@lanl.gov.