Velocity Distributions of Granular Gases with Drag and with Long-Range Interactions

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We study velocity statistics of electrostatically driven granular gases. For two different experiments: (i) non-magnetic particles in a viscous fluid and (ii) magnetic particles in air, the velocity distribution is non-Maxwellian, and its high-energy tail is exponential, $P(v) \sim \exp(-|v|)$. This behavior is consistent with kinetic theory of driven dissipative particles. For particles immersed in a fluid, viscous damping is responsible for the exponential tail, while for magnetic particles, long-range interactions cause the exponential tail. We conclude that velocity statistics of dissipative gases are sensitive to the fluid environment and to the form of the particle interaction.

PACS numbers: PACS numbers: 45.70.-n, 47.70.Nd, 81.05.Rm

Despite extensive recent studies, a fundamental understanding of the dynamics of granular materials still poses a challenge for physicists and engineers [1–3]. Remarkably, even dilute granular gases substantially differ from molecular gases. A series of recent experiments on granular gases, driven either mechanically [4–10] or electrostatically [11], reveals that the particle velocity distribution significantly deviates from the Maxwell-Boltzmann distribution law. In particular, the high-energy tail of the velocity distribution $P(v)$ is a stretched exponential

$$P(v) \sim \exp\left(-|v/v_0|^{\xi}\right) \tag{1}$$

with v_0 the typical velocity. The exponent $\xi = 3/2$ is observed for certain vigorous driving experiments [4, 11]. Non-Maxwellian velocity distributions were also observed in experiments with a variety of geometries and driving conditions [5–10] and in numerical simulations [12–18]. Energy dissipation is responsible for this behavior and this can be understood using a simple model: a thermally driven gas of inelastic hard spheres. For high-energy particles, there is a balance between loss due to inelastic collisions and gain due to the thermal driving. For hard-core interactions, kinetic theory predicts (1) with $\xi = 3/2$, in agreement with vigorous shaking experiments [19]. However, interactions between particles often do not reduce to simple hard-core exclusion.

In this Letter, we study the effects of fluid environment and particle interactions on electrostatically driven granular gases. We perform experiments with particles immersed in a viscous fluid and with magnetic particles in air subjected to an external magnetic field. We find that the high-energy tail of the velocity distribution is characterized by (1) but with the exponent $\xi = 1$. We generalize the kinetic theory to situations with viscous damping and with long-range interactions and find that the experimental results are in-line with the kinetic theory predictions. We conclude that velocity statistics in granular gases depend sensitively on the environment and on the form of the particle interaction.

Our experimental setup is similar to that in Ref. [20–22], see Inset to Fig. 1. The particles are placed between the plates of a large capacitor that is energized by a constant (dc) or alternating (ac) electric field $E = E_0 \cos(2\pi f t)$. To provide optical access to the cell, the capacitor plates were made of glass with a clear conductive coating. We used 11×11 cm capacitor plates with a spacing of 1.5 mm (big cell) or 4 cm diameter by 1.5 mm cell (small cell). The particles are 165 μm diameter non-magnetic bronze spheres or 90 μm magnetic nickel spheres. The field amplitude E_0 varied from 0 to 10 kV/cm and the frequencies f were between 0 and 120 Hz. The total number of particles in the cell is on the order 10^6. To control the magnetic interactions, the cell was placed inside a large 30 cm electromagnetic coil capable of creating dc/ac magnetic field H up to 80 Oe. The cell can be filled with non-polar dielectric fluid (toluene) to introduce viscous damping. The electro-cell works as follows: conducting particles acquire a surface charge when they are in contact with the capacitor plate. If the magnitude of the electric field exceeds gravity, particles travel upwards, recharge upon contact and then fall down. This process repeats in a cyclic manner. By applying ac electric field and adjusting its frequency f, one controls the vertical extent of particles motion by effectively turning them back before they collide with the upper plate, making the system effectively two-dimensional.

We extracted horizontal particle velocities using high-speed videomicroscopy. Images were obtained in transmitted light at a rate up to 2,000 frames per second from a camera mounted to a long focal distance microscope. Particle positions were determined to sub-pixel resolution. Inter-particle and particle-boundary collisions that introduce sudden changes in momenta were filtered out in a manner similar to Ref. [5]. An ensemble average for each of the velocity distributions was obtained from about $5 \cdot 10^6$ data points.

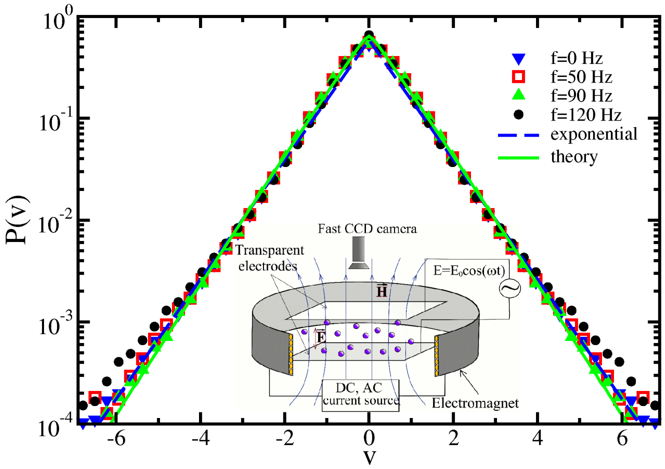

FIG. 1: Velocity distribution functions for particles immersed in fluid. The data are for 165 μm bronze particles immersed in fluid (toluene) at frequencies $f = 0, 50, 90, 120$ Hz and applied voltage $U = 950$V. Corresponding rms velocities $v_{\rm rms} = 1.68$, 1.27, 0.75, 0.70 cm/s. Dashed line shows best fit to pure exponential distribution $P(v) \sim \exp(-|v|/v_0)$ for $f = 90$ Hz, and solid line shows theoretical result from Eq. (2) with $\eta = 0.1$. Inset: Schematics of the experimental apparatus.

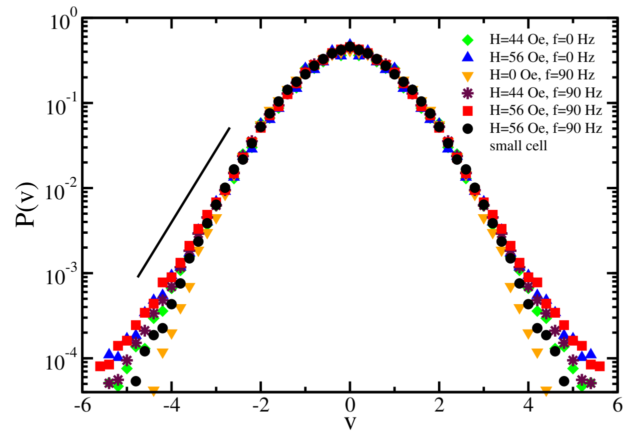

FIG. 2: Velocity distributions for magnetic particles. The data corresponds to 90 μm nickel spheres for dc and ac ($f = 90$ Hz) electric field, $U = 1000$V in 11×11 cm cell, in dc magnetic field. Corresponding rms velocities $v_{\rm rms}$ for $f = 0$ (dc) are 1.9 cm/s ($H = 44$ Oe) and 2.53 cm/s ($H = 44$ Oe), and for $f = 90$ Hz $v_{\rm rms} = 3.0, 2.12, 2.34$ for $H = 0, 44, 56$ Oe. Data for a small 3 cm cell is also shown for comparison. Exponential decay is indicated by straight line as a reference.

We performed two sets of experiments: (i) electrostatically driven non-magnetic particles in viscous fluid; (ii) electrostatically driven magnetic particles in air subjected to external magnetic field. Some experiments were also performed with magnetic particles in fluid. Although the origin of the particle interaction is very different, both systems happen to show somewhat similar behavior: exponential asymptotic velocity statistics. For the fluid system, the exponential behavior results mostly from the dominant viscous drag. However, the effects of hydrodynamic dipole-dipole interaction between moving particles in fluid are of certain importance: the hydrodynamic interaction between particles become comparable with the viscous drag if the particles are close enough or in contact [23]. This interaction has consequence for high velocity tail, see discussion below. The ratio of viscous drag force F_d to the gravity force F_g at rms velocity in toluene is about 0.2-0.3 and in air is less then 0.007. Thus, viscous drag effects are obviously dominant in toluene. For the magnetic system the exponential behavior is attributed to dominant long-range dipole interaction since the air drag is negligible. Simple estimates show that the magnetic dipole forces between particles dominate gravity if the distance is smaller than 3 particles diameters. Thus, due to remnant magnetization of the particles magnetic interaction is dominant even for $H = 0$.

Representative results for the fluid system are shown in Fig. 1. Pure toluene was used in most experiments. Further experiments were performed using a toluene/polysterene mixture in order to control the viscosity of the solution, but no qualitative differences were found. Throughout this Letter, we analyze the distribution of the horizontal velocity components, $P(v)$, with $v \equiv v_x, v_y$. The velocity is normalized such that the root-mean-square (rms) velocity equals one, $\langle v^2\rangle = 1$, and of course, the velocity distributions are symmetric, $P(v) = P(-v)$. As shown in Fig. 1, the velocity distributions are all notably different than the Maxwellian distribution. Moreover, the best fit to Eq. (1) gives the value $\xi = 1$ in a wide range of parameters (driving amplitude and frequency). Remarkably, the velocity distributions for fluid-filled cells are different from those obtained for air-filled cells (where viscous drag is negligible) with $\xi = 3/2$, other parameters the same [11].

Experiments with magnetic interactions were performed using nickel magnetic microparticles with an average size of about 90μm (Alfa Aesar Company). The magnetic moment per particle at 80 Oe is $1 \cdot 10^{-5}$ emu; the saturated magnetic moment is $2 \cdot 10^{-4}$ emu per particle and the saturation field is about 4 kOe. A vertically oriented external magnetic field was applied in order to control the magnetic interactions: since the particles are multi-domain, and nickel is a soft magnetic material, the applied field can effectively increase the particle's magnetic moment. The corresponding velocity distributions are shown in Fig. 2. The magnetic field systematically widens the velocity distribution and it enhances the exponential asymptotic decay of $P(v)$ at the high velocities. This observation is consistent with the fact that the applied magnetic field enhances the dipole-dipole magnetic interaction due to the magnetization of the particles. Comparing the fluid and the magnetic systems, we note that the velocity distributions have different cores, but the tails are exponential in both cases.

In the course of the measurements we noticed that finite size effects have a strong influence on the tail of $P(v)$. We performed a number of measurements using a 4 cm diameter cell, and about 3,000 particles. At the same conditions, the velocity distribution in the larger cell has a more pronounced exponential tail (see Fig. 2).

We also carried out several experiments combining features of the two systems using magnetic particles in toluene solution, and obtained results consistent with the rest of our observations: non-Maxwellian velocity distributions with an exponential tail. Compared with pure magnetic interactions results, the range of exponential behavior for the tail becomes even more extended.

We now compare the experimental results with the predictions of kinetic theory, the standard framework for describing granular gases. This requires generalization of (1) to situations with long-range interactions. For particles interacting via the potential $U(r) \sim r^{-\sigma}$, r is the interparticle distance, the collision rate K grows algebraically with the normal component of velocity difference Δv, $K \propto (\Delta v)^{\lambda}$ with $\lambda = 1 - 2\frac{d-1}{\sigma}$, and d is the dimension of space [24]. Hard-spheres, $\lambda = 1$, model granular particles with hard-core ($\sigma \equiv \infty$) interactions while Maxwell molecules [25], $\lambda = 0$, model granular particles with a specific dipole interaction. In two-dimensions, relevant to our experiments, the collision rate effectively becomes independent on the relative velocity when $\sigma = 2$.

First consider magnetic particles. To analyze the high-energy tail, we make the standard assumption that forcing is thermal. For energetic particles, gains due to collisions are negligible and losses due to collisions are balanced by the forcing. The high-energy tail of the velocity distribution is governed by this balance, $d^2P(v)/d^2v \propto K(v)P(v) \propto v^{\lambda}P(v)$ [27]. Consequently, the velocity distribution decays as a stretched exponential (1) with $\xi = 1+\lambda/2$. In general, the velocity statistics are non-Maxwellian and the tails are over-populated with respect to a Maxwellian distribution. The upper limit, $\xi = 3/2$, is realized for hard-spheres, and pure exponential behavior, $\xi = 1$, occurs for Maxwell molecules. For magnetic dipole interactions one has $\sigma = 2$. In this case, in two dimensions the kinetic theory predicts a simple exponential tail, $\xi = 1$.

In the following, we consider a representative data set for the fluid system (120 Hz) and for the magnetic system (44 Oe, 90 Hz). We display a single set because variations in $P(v)$ among the different experimental conditions are relatively small: the kurtosis $\kappa \equiv \langle v^4\rangle/\langle v^2\rangle^2$ varies by less than 5% for the different data sets.

For the fluid system, the velocity distribution is very close to a pure exponential, as shown in Fig. 1, and furthermore, the kurtosis $\kappa = 6.2 \pm 0.2$ is within 3% of the value corresponding to a pure exponential distribution $\kappa = 6$. Viscous damping is responsible for this behavior and the nearly exponential distribution is consistent with the damping process $v \to \eta v$ suggested by van Zon et al [16]. This mimics viscous damping because $v_n = v_0\eta^n$ [16] with n the number of damping events and n growing linearly with time. The damping rate is set by the frequency of collisions with the plates, but in the theory,

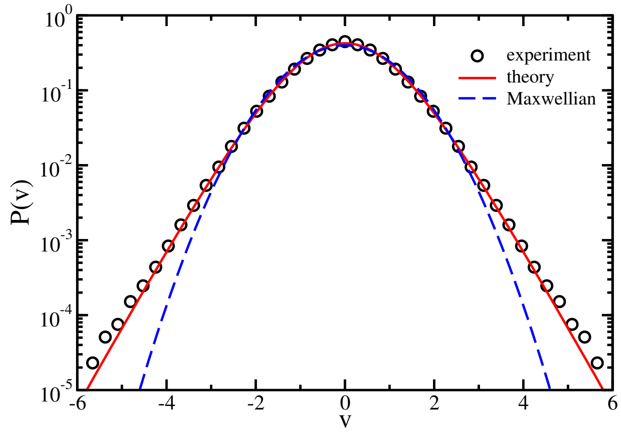

FIG. 3: The experimental distribution for magnetic particles vs the theory for forced Maxwell molecules. Crossover to exponential law occurs for $|v| > 2$.

it can be set to one without loss of generality. When the viscous dissipation dominates over the collisional dissipation, the kinetic theory is modified as follows

$$D\partial_v^2 P(v) + \eta^{-1}P\left(v/\eta\right) - P(v) = 0 \qquad (2)$$

where the last two terms represent gain and loss due to drag and D is diffusion coefficient. At large velocities, the gain term is negligible and consequently, the tail is exponential $P(v) \sim \exp\left(-v/\sqrt{D}\right)$. Eq. (2) can be solved analytically, and there is a family of velocity distributions characterized by one parameter η. When $\eta \to 0$, the gain term in (2) is negligible and the distribution is purely exponential. This is reflected by the kurtosis $\kappa = \frac{6}{1+\eta^2}$. For strong damping, $\eta \to 0$, the value corresponding to a pure exponential distribution is realized, $\kappa \to 6$.

A Discrete Simulation Monte Carlo methods was used to solve the Boltzmann equation for Maxwell molecules. In the simulations pairs of randomly chosen particles collide according to the inelastic collision rule $(\mathbf{v}_1, \mathbf{v}_2) \to (\mathbf{v}_1', \mathbf{v}_2')$ with $(\mathbf{v}_1' - \mathbf{v_2}') \cdot \hat{\mathbf{n}} = -\alpha(\mathbf{v_1} - \mathbf{v_2}) \cdot \hat{\mathbf{n}}$ and $\mathbf{v}_1' + \mathbf{v_2}' = \mathbf{v_1} + \mathbf{v_2}$ with α the restitution coefficient and $\hat{\mathbf{n}}$ the impact direction. In addition, particles are thermally forced $dv/dt = \zeta$ with ζ a white noise. Also, damping $v \to \eta v$ with unit rate models the fluid effect. The simulation results represent an average over 10^2 runs in a system with $N = 10^7$ particles.

When viscous damping dominates over collisional dissipation, the distribution is nearly exponential, see Fig. 1, and in a very good agreement with the experiments. We note that even though the drag term dominates over the collision terms, the experimental results suggest that the collision rate is velocity independent at least at large velocities, $\lambda = 0$. If this were not the case, the collisional loss term would dominate at some very large velocity and there would be a cross over to $\xi > 1$ [26]. But no such crossover is observed experimentally. We conclude that the results of kinetic theory of forced Maxwell molecules with strong viscous damping are consistent with the experimental results for all velocities.

For magnetic interactions, there is an excellent agreement between the experiment and the theory of thermally forced Maxwell molecules for which the collision rate is completely independent on the relative velocity (see Fig. 3). The kurtosis, $\kappa = 3.6 \pm 0.1$, falls within 2% of the analytically known value $\kappa = 3 + \frac{18}{33} \cong 3.55$ obtained from $\kappa = 3 + \frac{18p^2(1-p)}{(d+2)(1+p)-3(1-p)(1+p^2)}$ where $p = (1-\alpha)/2$ [27]. We note that there are no fitting parameters. The restitution coefficient α was set to zero because particle collisions in the experiments are strongly inelastic and because as long as the dissipation is strong, there is only a weak dependence on α. Even though the tail of the distribution is close to a simple exponential, its core is approximately Maxwellian, as reflected by the kurtosis that is much closer to the pure Maxwellian value of 3 than the pure exponential value of 6. We conclude that for magnetic particles the collision rate becomes practically independent on the relative velocity, and conversely, that they are accurately modeled by Maxwell molecules.

In summary, our main result is that velocity statistics of forced granular gases depend sensitively on the fluid environment and on the nature of the inter-particle interactions. The two sets of experiments can be universally described by a specific version of the kinetic theory, Maxwell molecules, with a velocity independent collision rate. Whereas dipole interactions are ubiquitous for the magnetic system, our studies indicate that the Maxwell model is the only way to interpret the fluid experiments. If hard sphere were used, the collision rate will grow linearly with the velocity and $\xi = 3/2$ stretched exponential tail will prevail at large velocities. No such crossover is observed and this is a strong evidence that the collision rate is velocity independent. Thus, although the core behavior is dominated by the damping, the tail behavior indicates that long range interactions play a role. In view of this, the two experiments are complementary.

We comment that it is difficult to experimentally validate that the driving is thermal in nature. However, the consistent agreement between the experiments and theory supports this widely-used modeling assumption. In addition, the excellent quantitative agreement between the magnetic particles experiments and the Maxwell molecules kinetic theory suggests that magnetic particles are an ideal experimental probe for the predictions of this analytically tractable theory, including in particular, the transport coefficients [28]. We also propose that stretched exponential velocity distributions may be generic for dissipative gases with competing interactions, and may possibly be relevant for vastly different systems, such as dusty plasmas, colloids, and even star clusters, where long-range interactions (e.g due to gravity) are mediated by short-range collisions.

We thank R. Brito and M. Ernst for useful discussions and acknowledge support from the US DOE, Office of Science, contracts W-31-109-ENG-38 (IA,AS) and W-7405-ENG-36 (EBN), the General Research Fund at the University of Kansas and Kansas NASA EPSCoR (JSO).

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